Investigation on Turbulence Effects on Flutter Derivatives of Suspended Truss Bridge Section

L. Hoang-Trong, V. Nguyen-Hoang and H. Vo-Duy

Abstract This paper presents the flutter derivatives extracted from a stochastic state-space system identification method under difference turbulent flows. The aim of the study is to clarify the effects of oncoming turbulence on the flutter of suspended long-span bridges by using section model wind tunnel test. Several wind tunnel tests on a trussed deck section have been carried out with different oncoming turbulent properties involving turbulence intensities and turbulent scales. The analysis includes the transient response data from wind tunnel test which have been analyzed by the system identification technique in extracting flutter derivatives (FDs) and the difficulties involved in this method. The time-domain analysis stochastic system identification is proposed to extract simultaneously all FDs from two degree of freedom systems. Finally, the results under different condition were discussed and concluded.

Keywords Flutter derivative · Stochastic system identification
Wind turbulence · Flutter critical velocity

1 Introduction

The wind in the atmospheric boundary layer is always turbulent. Therefore, any research of wind-induced vibration problems must consider this issue. Not many researches have focused clearly on the effects of turbulence on aeroelastic forces. Scanlan (1978) [1] is the pioneer who used a trussed deck section model and then concluded that flutter derivatives had an insignificant difference from smooth and turbulent flows. However, Huston (1986) conducted a test on a model of the Golden Gate Bridge deck section, and the results showed a significant discrepancy in flutter derivatives between smooth and turbulence flows [2]. Sarkar et al. (1994) [3]

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conducted a streamlined steel box girder section model test under both smooth and turbulence flows and applied robust system identification method, Modified Ibrahim Time Domain (MITD) [4]. Their study showed two valuable conclusions: (1) The turbulence flow does not appreciably affect self-excited forces via the FDs, and (2) the MITD method is based on the assumption that there is no external excitation of the system. For a section model immersed in turbulent flow, the buffeting forces and these responses are considered external forces, and hence free vibration condition is conflicted theoretically. The MITD method treats the resulting forced response as though additional noise is presented in the signals. This made the identification flutter derivatives more difficult and most likely reduced the accuracy.

For a study on FDs, the free vibration technique of sectional model is used, and system identifications (SID) technique to extract FDs is widely applied [3–5]. Various SID techniques were developed by many authors: the Extended Kalman Filter Algorithm, Modified Ibrahim Time-Domain method, Unifying Least-squares method, and Iterative Least-Squares method. In these systems, the buffeting force and their response are considered as external noise, so this causes more difficulties at high wind velocity such as noise increase due to turbulence.

Kirkegaard and Andersen [6] compared three state-space systems: stochastic subspace identification (SSI), stochastic realization estimator matrix block Hankel (MBH), and prediction error method (PEM). The results showed that the SSI gave good results in terms of estimated modal parameters and mode shapes. The MBH was found to give poor estimates of the damping ratios and the mode shapes compared with the other two techniques. In addition, the SSI was approximately ten times faster than the PEM.

This study is to clarify the effects of oncoming turbulence on self-excited force of a suspended long-span bridge deck. The more challenging is the application of a stochastic system identification method to identify flutter derivatives from free decay response for the section model which is obtained by an experimental wind tunnel test for a truss deck section. The output only time-domain analysis stochastic system identification, also known as data driven stochastic system (SSI-data) methods is proposed to extract simultaneously all flutter derivatives (FDs) from two degrees of freedom system (DOF).

2 Wind Tunnel Test

A wind tunnel test was conducted in a closed-circuit wind tunnel of Yokohama National University. The investigated profile is trussed deck section (Fig. 1). Since the truss deck with closed open grating exhibits torsional flutter at a relatively low wind speed, FDs (particularly A_2^*) identified can be validated by a flutter onset wind speed. The width and depth of the section model are 363 mm and 162.5 mm, respectively. The unit mass is 8.095 kg/m, and moment of inertia is 0.2281 kg m^2/m. The vertical frequency and damping ratio are 1.869 Hz and 0.0051, respectively.

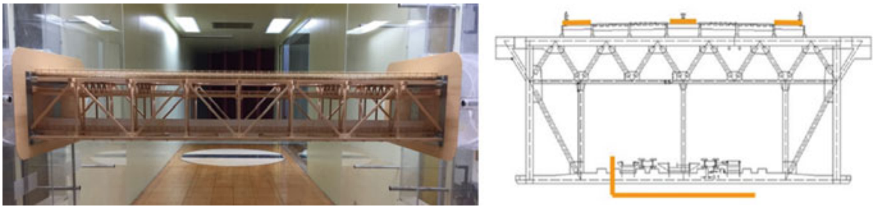


Fig. 1 Truss deck section model

Table 1 Turbulence intensity and scale

	I_u (%)	I_w (%)	L_u (cm)	L_w (cm)
Case 1	6.2	4.6	11.3	9.1
Case 2	9.1	6.9	9.0	8.7
Case 3	15.6	13.2	6.8	6.4

I turbulence intensity, *L* turbulence integral length scales with horizontal u and vertical direction w

The torsional frequency and damping ratio are 3.296 Hz and 0.00419, respectively. The tests have been carried out in both smooth and different turbulent flows. The turbulent flows used in this study are generated with biplane wooden grids. The turbulent properties are controlled by changing the distances to the model. The flow conditions and turbulence properties are shown in Table 1.

3 Identification of Flutter Derivatives

3.1 *Stochastic Discrete-Time State-Space System*

Considering a 2 DOF section model of bridge deck in turbulent flow, equation of motion is written by

$$\begin{aligned} m[\ddot{h} + 2\xi_h\omega_h \dot{h} + \omega_h^2 h] &= L_{se} + L_b \\ I[\ddot{\alpha} + 2\xi_\alpha\omega_\alpha \dot{\alpha} + \omega_\alpha^2 \alpha] &= M_{se} + M_b \end{aligned} \quad (1)$$

where h and α are the vertical and torsional displacement; m and I are mass and mass moment of inertia per unit length, respectively; $\omega_h = 2\pi f_h$ and $\omega_a = 2\pi f_a$ are circular frequencies of heaving and pitching mode; ξ_h and ξ_a are damping ratio to critical; L_{se} and M_{se} are the aerodynamic self-excited lift and moment, respectively, given by Simiu and Scanlan [7]

$$\begin{aligned} L_{se} &= \frac{1}{2}\rho U^2 B\left[K_h H_1^*(K_h)\frac{\dot{h}}{U} + K_\alpha H_2^*(K_\alpha)\frac{B\dot{\alpha}}{U} + K_\alpha^2 H_3^*(K_\alpha)\alpha + K_h^2 H_4^*(K_h)\frac{h}{B}\right] \\ M_{se} &= \frac{1}{2}\rho U^2 B^2\left[K_h A_1^*(K_h)\frac{\dot{h}}{U} + K_\alpha A_2^*(K_\alpha)\frac{B\dot{\alpha}}{U} + K_\alpha^2 A_3^*(K_\alpha)\alpha + K_h^2 A_4^*(K_h)\frac{h}{B}\right] \end{aligned} \quad (2)$$

where ρ is the air density; U is the mean wind velocity; B is the width of bridge deck; $K_i = \omega_i B/U$ the reduced frequency ($i = h, \alpha$); H_i^* and A_i^* (i = 1, 2, 3, 4) are the flutter derivatives. L_b and M_b (factorized into matrix B_2 and input vector $u(t)$) are the buffeting forces in the vertical and torsional directions, respectively. By substituting L_{se}, M_{se}, L_b, and M_b into Eq. (1) and moving the aerodynamic damping and stiffness terms to the left-hand side, Eq. (1) can be transformed into a first-order state equation by Eq. (3) [8]

$$\dot{x}(t) = \begin{bmatrix} 0 & I \\ -[M]^{-1}[K^e] & -[M]^{-1}[C^e] \end{bmatrix}_{4x4} x(t) + \begin{bmatrix} 0 \\ [M]^{-1}B_2 \end{bmatrix} u(t) \quad (3)$$

where $x(t)$ is state vector; $[M]$ is mass matrix; $[C^e]$ is gross damping matrix including the structural damping and aerodynamic damping; $[K^e]$ is gross stiffness matrix including the structural stiffness and aerodynamic stiffness.

In the modal analysis, sometimes the input is unknown and measurements are mostly sampled at discrete-time. On the other hand, it is impossible to measure all DOFs, and measurements always have disturbance effects. For all these reasons, the continuous deterministic system will be converted to suitable form, discrete-time stochastic state-space model, as follows:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k \end{aligned} \quad (4)$$

where $x_k = x(k\Delta t)$ is the discrete-time state vector containing the discrete sample displacement and velocity; w_k is the process noise; v_k is the measurement noise due to sensor inaccuracy. Assuming w_k and v_k are zero mean and $\{x_k\}$, $\{w_k\}$, and $\{v_k\}$ are mutually independent, the output covariance $R = \mathrm{E}[y_{k+\mathrm{i}}\ y_k^{\mathrm{T}}]$ for any arbitrary time lags $i\Delta t$ can be considered as impulse response (Eq. (5)) of the deterministic linear time-invariance system A, C, and G, where $G = \mathrm{E}[x_{k+1}\ y_k^{\mathrm{T}}]$ is the next state-output covariance matrix.

$$R_i = CA^{i-1}G \quad (5)$$

3.2 Stochastic System Identification (SSI)

Data-driven stochastic subspace identification (SSI-data) method [9] is used in this study. It works directly with time series of experimental data without the need to convert output data to correlation, covariance, or spectra. The main step of SSI-data is a projection of the row space of the future outputs into the row of past outputs. The orthogonal projection P_i is defined as follows:

$$P_i = Y_f / Y_p = Y_f Y_p (Y_p Y_p^T)^{-1} Y_p \tag{6}$$

where the matrix Y_f and Y_p are the under half part and upper part half of a block Hankel matrix H as below.

$$H = \begin{bmatrix} y_0 & y_1 & \cdots & y_{j-1} \\ y_1 & y_2 & \cdots & y_j \\ \cdots & \cdots & & \cdots \\ y_{i-1} & y_i & \cdots & y_{i+j-2} \\ y_i & y_{i+1} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & \cdots & y_{i+j} \\ \cdots & \cdots & & \cdots \\ y_{2i-1} & y_{2i} & \cdots & y_{2i+j-2} \end{bmatrix}_{2ixj} = \left[\frac{Y_{0|i-1}}{Y_{i|2i-1}}\right] = \left[\frac{Y_p}{Y_f}\right] \frac{\updownarrow li}{\updownarrow li} \tag{7}$$

where y = (y_0, y_1, y_2, y_2… y_n) $\in R^{lxn}$ is the output measurement data obtained from l sensors (in this study $l = 2$ for heaving and torsion modes), $2i$ is the number of block rows, and j is the number of columns.

The main theorem of stochastic subspace identification states that the projection P_i can be factorized as the product of observability matrix O_i and the Kalman filter state sequence $\hat{X}_i$. The observability matrix O_i and the Kalman filter sequence $\hat{X}_i$ are obtained by applying SVD to the projection matrix P_i. The Kalman state sequences $\hat{X}_i$, $\hat{X}_{i+1}$ are calculated using only output data. The system matrices can now be recovered from overdetermined set of linear equations and obtained by extending Eq. (4)

$$\begin{pmatrix} \widehat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} = \begin{pmatrix} A \\ C \end{pmatrix} \left(\widehat{X}_i\right) + \begin{pmatrix} \rho_w \\ \rho_v \end{pmatrix} \tag{8}$$

where $Y_{i/i}$ is a Hankel matrix with only one block row. Since the Kalman state sequence and the outputs are known and the residuals $(\rho_w^T \ \rho_v^T)^T$ are uncorrelated with $\hat{X}_i$, the set of equation can be solved for A and C in the least-squares, where $(.)^u$ is pseudo-invert of a matrix

$$\begin{pmatrix} A \\ C \end{pmatrix} = \begin{pmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{pmatrix} \left(\hat{X}_i\right)^{\dagger} \quad (9)$$

3.3 Identification of Flutter Derivatives

The modal parameters of system can be obtained by solving the eigenvalue problem state matrix A as

$$A = \Psi \Lambda \Psi^{-1}, \quad \Phi = C\Psi \quad (10)$$

where Ψ is the complex eigenvector; Λ the complex eigenvalue is the diagonal matrix; Φ the mode shape matrix. When the complex modal parameters are known, the gross damping C^e and gross stiffness K^e in Eq. (3) are determined by following:

$$[K^e \quad C^e] = -M\left[\Phi\Lambda^2 \quad \Phi^*(\Lambda^*)^2\right]\begin{bmatrix} \Phi & \Phi^* \\ \Phi\Lambda & \Phi^*\Lambda^* \end{bmatrix}^{-1} \text{ and } \begin{cases} \overline{C}^e = M^{-1}C^e; & \overline{K}^e = M^{-1}K^e \\ \overline{C} = M^{-1}C^0; & \overline{K} = M^{-1}K^0 \end{cases} \quad (11)$$

where C^0 and K^0 are the mechanical damping and stiffness matrix of the system under no-wind condition.

Thus, the flutter derivatives of 2 DOF can be defined as follows:

$$\begin{aligned}
H_1^*(K_h) &= -\frac{2m}{\rho B^2\omega_h}(\overline{C}_{11}^e - \overline{C}_{11}), & H_2^*(K_\alpha) &= -\frac{2m}{\rho B^3\omega_\alpha}(\overline{C}_{12}^e - \overline{C}_{12}) \\
H_3^*(K_\alpha) &= -\frac{2m}{\rho B^3\omega_\alpha^2}(\overline{K}_{12}^e - \overline{K}_{12}), & H_4^*(K_h) &= -\frac{2m}{\rho B^3\omega_h^2}(\overline{K}_{11}^e - \overline{K}_{11}) \\
A_1^*(K_h) &= -\frac{2I}{\rho B^3\omega_h}(\overline{C}_{21}^e - \overline{C}_{21}), & A_2^*(K_\alpha) &= -\frac{2I}{\rho B^4\omega_\alpha}(\overline{C}_{22}^e - \overline{C}_{22}) \\
A_3^*(K_\alpha) &= -\frac{2I}{\rho B^4\omega_\alpha^2}(\overline{K}_{22}^e - \overline{K}_{22}), & A_4^*(K_h) &= -\frac{2I}{\rho B^4\omega_h^2}(\overline{K}_{21}^e - \overline{K}_{21})
\end{aligned} \quad (12)$$

4 Flutter Derivatives

The decay responses are acquired at a sampling frequency 100 Hz, and these samples are set to zero before operating with MATLAB (Fig. 2).

The actual implementation of SSI-data consists of projecting (P_i) the row space of the under part outputs (Y_f) into the row space of the upper part outputs (Y_p) by applying robust numerical techniques QR factorization Eq. (6) and shifted projecting matrix P_{i-1}, computing SVD of P_i, truncating the SVD into the model order.

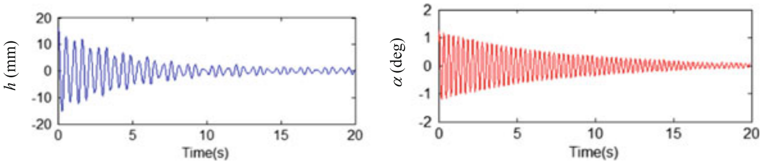


Fig. 2 Free decay response (V = 2.91 m/s) of the bridge deck section model (h-vertical; α-torsional)

The Kalman filter state sequence $\hat{X}_i$ is calculated by P_i then the SVD is found out. Continously, the state matrix A and C is obtained by least-square solution Eq. (9). The system order n can be determined from the number nonzero singular values of projecting matrix. In practice, affect by noise thus singular values that are all different from zero. Therefore, it is suggested to look at the "gap" between two successive singular values. The order will be selected by a maximum number of singular values at "gap" occur. Finally, the flutter derivatives will be obtained by comparing the gross damping and gross stiffness with mechanical damping and mechanical stiffness Eq. (12).

4.1 The Effects of Turbulence on Flutter Derivatives

In order to clarify the effects of oncoming flow turbulence on FDs, the SSI-data method is applied to extract FDs from free decay response with different turbulence intensity. Figure 3 shows the damping ratio of heaving and torsional mode versus reduced wind speed (V_r). Compared with smooth flow, the damping ratio of heaving mode increases more slowly, at certain reduced velocity; torsional damping ratio decreases when turbulence intensity increases.

Figures 4 and 5 show the FDs under smooth and turbulent flows with the different turbulence intensity versus reduced wind speed. H_1^*, H_4^*, A_1^*, A_4^* associated with vertical oscillation are identified using the vertical frequency, and

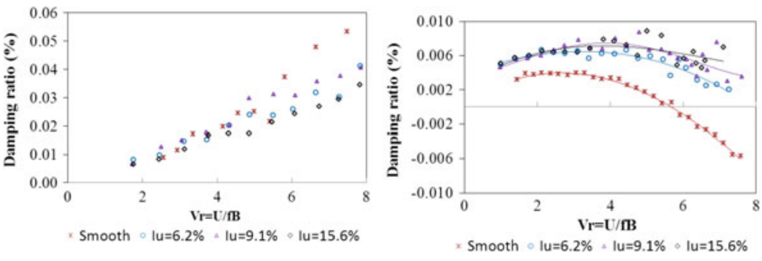


Fig. 3 Damping ratio of heaving (left) and torsional (right) mode of the bridge section model under smooth and turbulence flows (solid curves are fitted polynomial)

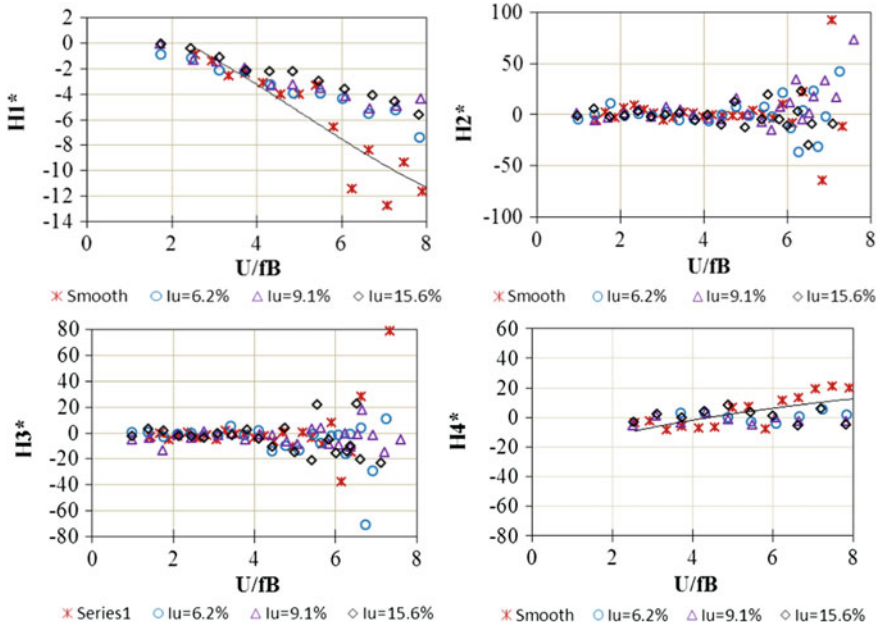


Fig. 4 FDs (H_i) of the bridge section model under smooth and turbulent flows by free decay response (solid curves are fitted polynomial of smooth case)

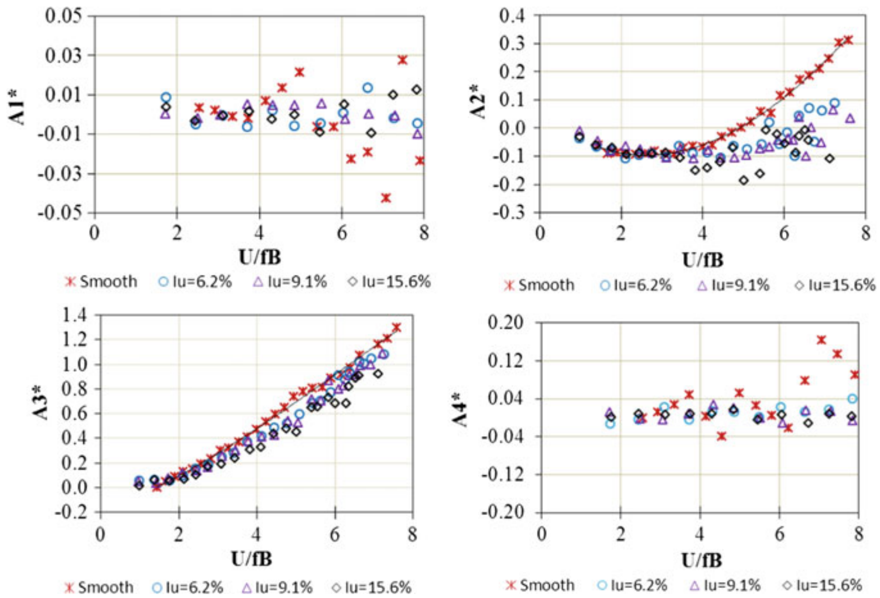


Fig. 5 FDs (A_i) of the bridge section model under smooth and turbulent flows by free decay response (solid curve is fitted polynomial of smooth case)

H_2^*, H_3^*, A_2^*, A_3^* associated with torsional oscillation are calculated using torsional frequency. The torsional damping term A_2^* plays an important role in torsional flutter instability since its positive/negative value corresponds to the aerodynamic instability/stability of torsional fluter. On the other hand, the coupled term H_3^* and A_1^* together with the aerodynamically uncoupled term A_2^* has significant role on heaving-torsional 2 DOF-coupled flutter instability.

From Figs. 4 and 5, it can be found that in smooth flow, the FD H_1^* increases faster than that extracted from turbulent flows. This means that damping ratio of heaving mode under smooth flow is higher compared with that from turbulent flow. Turbulence has a very small effect on vertical and torsional frequency terms H_4^* and A_3^*.

In this experiment, the onset flutter is defined zero cross of reduced velocity axis with the A_2^*. Under smooth flow, the positive value A_2^* at reduced wind speed ($V_r = 5.2$) coincides with the negative total torsional damping. The significant effects of turbulence flows on flutter derivatives are also illustrated particularly for aerodynamic torsional damping term A_2^*, the positive value correspond to the V_r around 6.5 to 7.8 under $I_u = 6.2\%$ and $I_u = 9.1\%$, respectively, whereas in case of $I_u = 15.6\%$, flutter does not occur up to $V_r = 8$. On the other hand, the effects of different turbulent intensities on FDs are fairly modest. Slight difference can be seen that A_2^* tends to be lower in a high reduced velocity range as turbulence intensity increases. The influence of turbulence on FDs will depend on the section. Sarkar (1994) [4] found small effect for a streamlined section, while tests on a truss section showed appreciable effect which is shown clearly by torsional damping term A_2*.

The off-diagonal terms H_2*, H_3*, A_1*, and A_4* are fluctuated around zero value, which means that in this experiment, the coupled vibration does not appear at small wind velocity.

4.2 Flutter Critical Velocity

In order to confirm results of identified FDs under free decay responses, the flutter critical velocity will be obtained from an equation of motion of a 2DOF system [10]

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = [F]\{\ddot{u}\} \tag{13}$$

where

$$[M] = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \quad [C] = \begin{bmatrix} 2m\xi_h\omega_h & 0 \\ 0 & 2I\xi_\alpha\omega_\alpha \end{bmatrix} \quad [K] = \begin{bmatrix} m\omega_h^2 & 0 \\ 0 & I\omega_\alpha^2 \end{bmatrix}$$

$$[F] = \begin{bmatrix} L_h & L_\alpha \\ M_h & M_\alpha \end{bmatrix} \quad \{u\} = \begin{Bmatrix} h \\ \alpha \end{Bmatrix}$$

For stability check, self-excited force is only considered, and L_z, L_θ, M_z, and M_θ are self-excited force components defined by

$$\begin{aligned} L_h &= -\pi\rho B^2(L_{hR} + iL_{hI}),\ L_\alpha = -\pi\rho B^2(L_{\alpha R} + iL_{\alpha I}) \\ M_h &= -\pi\rho B^4(M_{hR} + iM_{hI}),\ M_\alpha = -\pi\rho B^4(M_{\alpha R} + iM_{\alpha I}) \end{aligned} \tag{14}$$

where L_{hR}, L_{hI}, $L_{\alpha R}$, $L_{\alpha I}$, M_{hR}, M_{hI}, $M_{\alpha R}$, and $M_{\alpha I}$ are self-excited force coefficients (flutter derivatives) those can be compared with those by Scanlan's format as follows:

$$\begin{aligned} L_{hR} = H_4^*/2\pi,\ L_{hI} = H_1^*/2\pi,\ L_{\alpha R} = H_3^*/2\pi,\ L_{\alpha I} = H_2^*/2\pi \\ M_{hR} = A_4^*/2\pi,\ M_{hI} = A_1^*/2\pi,\ M_{\alpha R} = A_3^*/2\pi,\ M_{\alpha I} = A_2^*/2\pi \end{aligned} \tag{15}$$

Flutter derivatives of truss bridge deck section given in this study are approximated polynomials of the results from Figs. 4 and 5.

Assuming sinusoidal motion $\{u\} = \{u_0\}\exp(i\omega t)$ and since structural damping of a long-span bridge can be negligibly small, the damping matrix in Eq. (13) can be dropped. Then, the aerodynamically influenced equation of motion can be written by

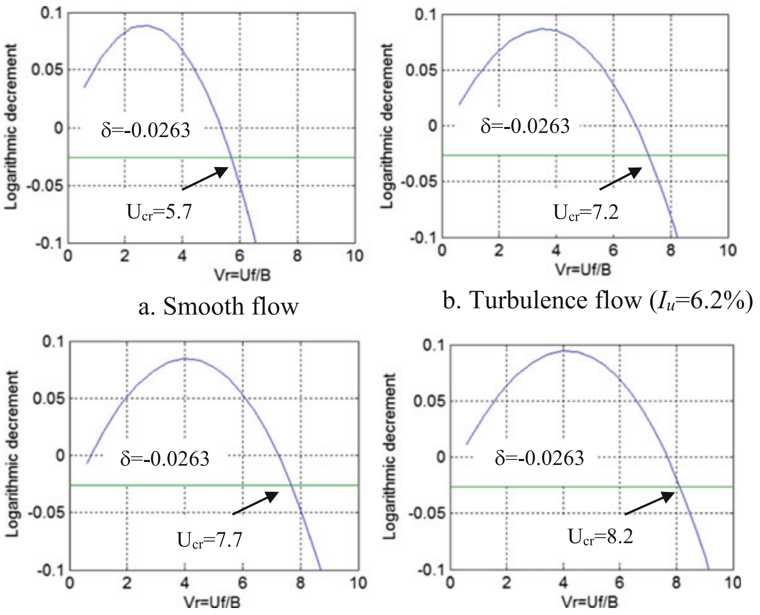
$$[K]^{-1}([M] - [F])\{\ddot{u}\} = \frac{1}{\omega^2}\{\ddot{u}\} \tag{16}$$

Solving Eq. (16) as an eigenvalue (λ) problem gives the modal frequency and modal damping ratio as Eq. (17).

$$\begin{aligned} \omega_i &= \sqrt{\mathrm{Re}(\lambda_i)^2 + \mathrm{Im}(\lambda_i)^2} \\ \zeta_i &= \mathrm{Re}(\lambda_i)/\sqrt{\mathrm{Re}(\lambda_i)^2 + \mathrm{Im}(\lambda_i)^2} \end{aligned} \tag{17}$$

where λ is an eigenvalue, Re(λ) and Im(λ) are real and image parts of eigenvalue, respectively.

The stability condition of the system is estimated based on modal damping ratio (or logarithmic decrement). Figure 6 shows the change of aerodynamic damping of torsional mode. The flutter critical wind speed (U_{cr}) is defined by the cross point of torsional aerodynamic logarithmic decrement and equivalent torsional structural logarithmic decrement ($\delta = -0.0263$). The flutter critical velocity increases with increase of turbulence intensity (Fig. 6). In the case of smooth flow the $U_{cr} = 5.7$. And $U_{cr} = 7.2$, $U_{cr} = 7.7$ and $U_{cr} = 8.2$ correspond to $I_u = 6.2\%$, $I_u = 9.1\%$ and $I_u = 15.6\%$, respectively.



a. Smooth flow b. Turbulence flow (I_u=6.2%)

c. Turbulence flow (I_u=9.1%) d. Turbulence flow (I_u=15.6%)

Fig. 6 Change of total logarithms decrement and flutter critical velocity

5 Conclusions

This study has investigated the effects of turbulence on flutter derivatives of the truss bridge deck section by using wind tunnel test and output only state-space stochastic system identification technique. Conclusions from this study are summarized as follows:

- SSI-data shows good results because of an advantage of the method considered buffeting force and its response as inputs instead of noise
- Turbulent flow significantly affects self-excited force via FDs of the truss bridge deck section. A_2^* became positive at $V_r = 5.2$ in the smooth flow and delayed to $V_r = 6.5$–7.8 in the turbulent flows of $I_u = 6.2\%$ and 9.1%, respectively; and in case of $I_u = 15.6\%$, positive value did not appear.
- Turbulence induces buffeting response but increases flutter critical velocity.

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